# Capacitive Reactance

**by Larry R. Luchi W7KZE**

*[Editor's Note: This technical discussion of capacitive reactance is a bit more advanced than what we usually publish in* Radio Fun, *and it may leave some of you in the dark. If so, read it through a few times—slowly—and things should begin to sink in. If you want to get the most out of amateur radio, you've got to really understand this stuff—not just memorize a bunch of answers for a test. We've asked Mr. Luchi to prepare a series of articles focusing on various electronic components and their properties. We'll publish them from time to time, and if you stick with them, you'll be well on your way to a solid foundation in electronics.—David N1GPH]*

In the May issue of *Radio Fun* I discussed capacitance. In this issue we will look at *capacitive reactance*, the opposition that a capacitor has to alternating current or a varying current. Symbolized $X_C$ and pronounced X sub C, this opposition to flow is different from ohmic resistance because the opposition depends on the charging and discharging of a capacitor rather than atomic collisions within the material. Since it takes time to charge a capacitor, $X_C$ is frequency dependent. That's why current into an ideal capacitor leads the voltage across the capacitor by 90 degrees. This is exactly opposite the effect in an ideal inductor where the voltage leads the current by 90 degrees. Capacitive reactance is measured in ohms.

### Basic Ideas

An uncharged capacitor acts like a short circuit, whereas a fully-charged capacitor acts like an open circuit. When the current is sinusoidal (a periodic waveform that follows the sine function or an AC sine wave is considered to be the perfect waveform), the capacitor charges and discharges continuously during the cycle. Because of this, a capacitor has a different effect on alternating current than on direct current.

### Effect on Direct Current

Figure 1a shows a DC source connected to a capacitor and resistor in series. If the capacitor is fully charged, its voltage drop equals the source voltage, which implies no voltage drop across the resistor and no current in the circuit. In other words, a fully-charged capacitor blocks direct current. This assumes an ideal capacitor, one whose leakage current is zero. (Leakage current is extremely small direct current through the dielectric. You can ignore leakage current in most capacitors, but it may be significant in electrolytic capacitors.)

### Current

Figure 1b shows the same circuit with an AC source. Because the current is alternating, the capacitor will charge and discharge during each cycle. As a result, an alternating current does exist in the circuit. The amount of the AC current depends on the source voltage, frequency, and the capacitance. In Figure 1b, the AC current is the same throughout the circuit just as in any series circuit.

When the capacitance increases, the alternating current increases. If the capacitance decreases, the alternating current decreases. The point is that an ideal capacitor allows alternating current to pass, but blocks direct current. The larger the capacitance, the smaller the opposition to the alternating current.

### Definition of Capacitive Reactance

*Capacitive reactance* is the opposition that a capacitor has to the alternating current into it. For an ideal capacitor, the capacitive reactance is mathematically defined as the ratio of the rms (root-mean-square: sine wave rms = peak value multiplied by 0.707 (cos 45 degrees) voltage across a capacitor to the rms current into the capacitor:

Equation 1: $$X_C = \frac{V_C}{I_C}$$

where $X_C$ = capacitive reactance
$V_C$ = rms voltage across capacitor
$I_C$ = rms current into capacitor

Equation 1 is also valid for peak values and for peak-to-peak values. If you use an oscilloscope, peak-to-peak values are the most convenient to measure.

### Definition of K

This is the measurement of the ability of material to use the magnetic tendencies of electricity. Air has a $K_\varepsilon$ of 1, while mica has an average of 6. The larger the $K_\varepsilon$, the more capacitance.

### How C and f Effect $X_C$

Figure 2a shows an AC circuit with a rms current of 10 mA into the capacitor. If a dielectric with a $K_\varepsilon$ of 10 is used (Figure 2b), the rms current is 10 times larger because the capacitive reactance is now 10 times smaller. The dielectric increased the capacitance by a factor of 10, which allows 10 times as much current to flow. Another way to get less capacitive reactance is shown in Figure 2c. The frequency is 10 times higher than in Figure 2a. The effect is to increase the rms current by a factor of 10.

### Reactance Formulas

The basic formula for a capacitor is:

$$i = C \frac{dv}{dt}$$

This says that the charging current into a capacitor equals the capacitance times the instantaneous rate of voltage change.

### Calculating $X_C$

When the voltage is sinusoidal, calculus can be used to derive the following formula for capacitive reactance:

Equation 2: $$X_C = \frac{1}{2\pi fC}$$

This formula for capacitive reactance is useful when you know the values of frequency and capacitance. Notice that $X_C$ is inversely proportional to f and C. For example, if either f or C doubles in value, $X_C$ will be half of its former value.

Direct current does not change directions, it has no frequency. This would be equivalent to f = 0. To calculate the capacitive reactance for direct current, you can substitute f = 0 in to equation 2-2. The results in $X_C = \infty$, which means a capacitor presents an open circuit to direct current.

### Calculating C

Equation 2 contains three variables: $X_C$, f, and C. Given any two of these quantities, you can solve for the third. Another example is, if you know the values of f and C, you can substitute into the formula and solve for $X_C$.

When you know the values of f and $X_C$, you can solve for C by rearranging the equation to get:

$$C = \frac{1}{2\pi X_C}$$

### Calculating f

Another example is when you know the value of $X_C$ and C, but not f. In this case, the equation can be rearranged as:

$$f = \frac{1}{2\pi CX_C}$$

After substituting for C and $X_C$, you can solve for f. Memorize Equation 2. The other two formulas are easily derived from it.

When using Formula 2 with a calculator, use the [EE] or [EXP] key for the exponent (power of 10). Your calculator will then default to a positive exponent. The sign change key [+/-] will change to a negative exponent for microfarads, $10^{-6}$ or picofarads, $(10^{-12})$. Practice with the following examples.

**EXAMPLE for $X_C$ where Capacitance = 100 pF, Frequency = 1 MHz:**

$$X_C = \frac{1}{2\pi fC}$$

| ENTER | DISPLAY |
|---|---|
| 6.28 [X] 1 [EXP] 6 [=] | 6280000 |
| [X] 100 [EXP] [+/-] 12 [=] | 0.000628 |
| [1/X] | 1592.356688 $X_C = 1592\Omega$ |

**EXAMPLE 2: where Capacitance = 8 pF, Frequency = 28.1 MHz:**

| ENTER | DISPLAY |
|---|---|
| 6.28 [X] 28.1 [EXP 6 [=] | 176468000. |
| [X] 8 [EXP] [+/-] 12 [=] | 0.001411744 |
| [1/X] | 708.3437224 or $X_C = 708\Omega$ |

In our next article we will look at inductive reactance. RF

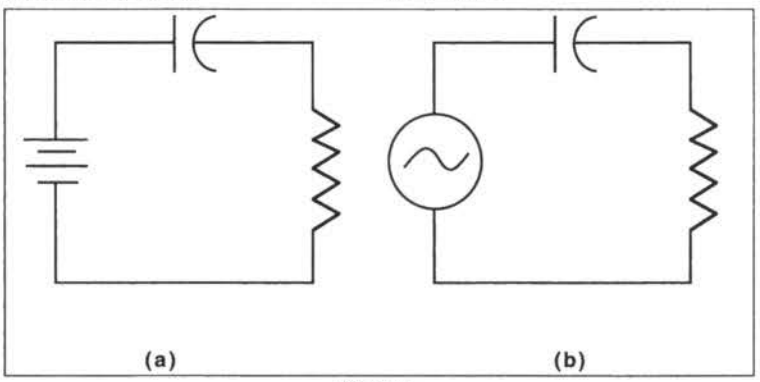


Figure 1.

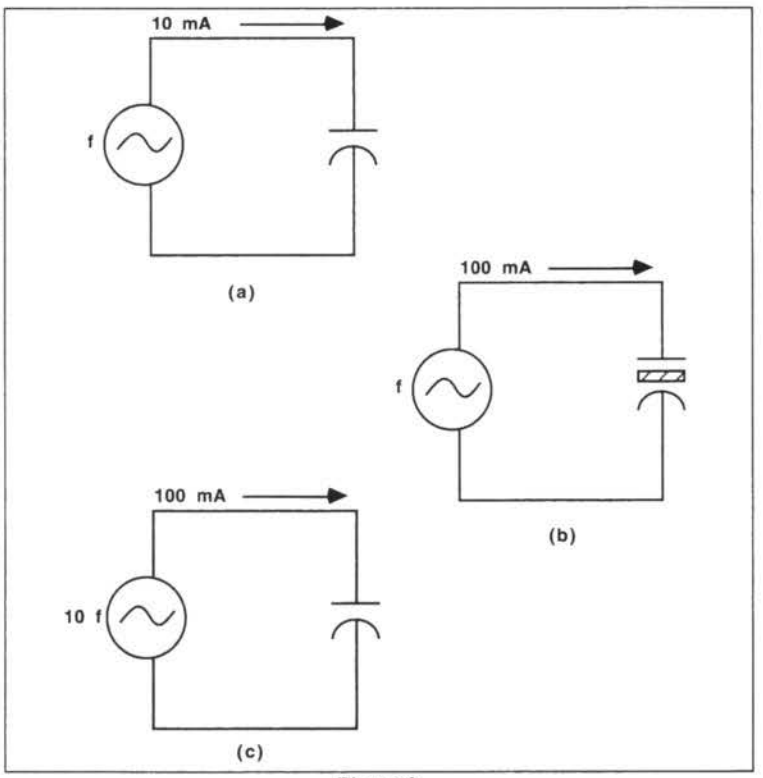


Figure 2.